

NDA-FP-73

April 2000

BRANE WORLD INFLATION INDUCED BY QUANTUM EFFECTS

SHIN'ICHI NOJIRI¹, SERGEI D. ODINTSOV^{♠2},

*Department of Applied Physics
National Defence Academy, Hashirimizu Yokosuka 239, JAPAN*

*♠ Instituto de Fisica de la Universidad de Guanajuato
Apdo.Postal E-143, 37150 Leon, Gto., MEXICO*

ABSTRACT

We consider brane-world universe where an arbitrary large N quantum CFT is living on the domain wall. This corresponds to implementing of Randall-Sundrum compactification within the context of AdS/CFT correspondence. Using anomaly induced effective action for domain wall CFT the possibility of self-consistent quantum creation of 4d de Sitter wall Universe (inflation) is demonstrated. In case of maximally SUSY Yang-Mills theory the exact correspondence with radius and effective tension found by Hawking-Hertog-Reall is obtained. The hyperbolic wall Universe may be induced by quantum effects only for exotic matter (higher derivatives conformal scalar) which has unusual sign of central charge.

¹email: nojiri@cc.nda.ac.jp

² On leave from Tomsk State Pedagogical University, RUSSIA.
email: odintsov@ifug5.ugto.mx

The idea [1] that we live on the brane (where four-dimensional gravity is recovered) embedded in higher-dimensional spacetime initiated enormous activity in the study of brane worlds. The attempts to realize the inflationary brane-worlds have been made in refs.[2, 3, 4, 5, 6, 7, 8] (and refs.therein). However, there is arbitrariness here. It is caused by the fact that details of scenario depend on the matter content (equation of state), etc. The inflationary brane-world scenario realized due to quantum effects of brane matter looks more attractive and universal. Such idea has been recently suggested in refs.[10, 9] (see also[11]) where addition of conformal quantum brane matter to complete effective action has been made. That corresponds to implementing of RS compactification within the context of renormalization group flow in AdS/CFT set-up. In particular, using conformal anomaly induced effective action the indication to a possibility of quantum creation of de Sitter or Anti-de Sitter wall in 5d AdS Universe has been demonstrated in ref.[10]. As brane quantum matter the maximally SUSY Yang-Mills theory on the wall has been considered. In ref.[9] the role of conformal anomaly in inducing of effective tension which is responsible for a de Sitter geometry of domain wall has been stressed and the corresponding effective tension has been calculated. The extensive calculation of (lorentzian) graviton correlator has been also made there. It was shown that domain wall CFT may significantly suppress the metric perturbations.

In the present Letter we give the details of such proposal (addition of brane quantum matter to RS scenario) with applications to 4d cosmology in the elegant form, using anomaly induced effective action. As a result one can consider the arbitrary content of CFT living on the wall. Moreover, the formalism is applied not only to 4d de Sitter wall but also to 4d hyperbolic wall in 5d Anti-de Sitter Universe or 4d conformally flat Universe. The equivalence of two approaches [10, 9] is explicitly proved: in case of spherical geometry the same effective tension and same equation for radius of de Sitter wall Universe is obtained. We also show that hyperbolic wall Universe may be realized due to quantum effects only for exotic matter (higher derivatives scalar). Note that consideration of large N conformal field theory living on the domain wall justifies such approach to brane-world quantum cosmology as then quantum contribution is essential.

We consider the spacetime whose boundary is 4 dimensional sphere S_4 , which can be identified with a D3-brane. The bulk part is given by 5 dimen-

sional Euclidean anti de Sitter space AdS_5

$$ds_{\text{AdS}_5}^2 = dy^2 + \sinh^2 \frac{y}{l} d\Omega_4^2. \quad (1)$$

Here $d\Omega_4^2$ is given by the metric of S_4 with unit radius. One also assumes the boundary (brane) lies at $y = y_0$ and the bulk space is given by gluing two regions given by $0 \leq y < y_0$.

We start with the action S which is the sum of the Einstein-Hilbert action S_{EH} , the Gibbons-Hawking surface term S_{GH} , the surface counter term S_1 and the trace anomaly induced action W^3 :

$$S = S_{\text{EH}} + S_{\text{GH}} + 2S_1 + W \quad (2)$$

$$S_{\text{EH}} = \frac{1}{16\pi G} \int d^5x \sqrt{g_{(5)}} \left(R_{(5)} + \frac{12}{l^2} \right) \quad (3)$$

$$S_{\text{GH}} = \frac{1}{8\pi G} \int d^4x \sqrt{g_{(4)}} \nabla_\mu n^\mu \quad (4)$$

$$S_1 = -\frac{3}{8\pi G} \int d^4x \sqrt{g_{(4)}} \quad (5)$$

$$\begin{aligned} W = & b \int d^4x \sqrt{\tilde{g}} \tilde{F} A \\ & + b' \int d^4x \left\{ A \left[2\Box^2 + \tilde{R}_{\mu\nu} \tilde{\nabla}_\mu \tilde{\nabla}_\nu - \frac{4}{3} \tilde{R} \Box^2 + \frac{2}{3} (\tilde{\nabla}^\mu \tilde{R}) \tilde{\nabla}_\mu \right] A \right. \\ & + \left(\tilde{G} - \frac{2}{3} \Box \tilde{R} \right) A \Big\} \\ & - \frac{1}{12} \left\{ b'' + \frac{2}{3} (b + b') \right\} \int d^4x \left[\tilde{R} - 6\Box A - 6(\tilde{\nabla}_\mu A)(\tilde{\nabla}^\mu A) \right]^2. \quad (6) \end{aligned}$$

Here the quantities in the 5 dimensional bulk spacetime are specified by the suffices $_{(5)}$ and those in the boundary 4 dimensional spacetime are by $_{(4)}$. The factor 2 in front of S_1 in (2) is coming from that we have two bulk regions which are connected with each other by the brane. In (4), n^μ is the unit vector normal to the boundary. In (6), one chooses the 4 dimensional boundary metric as

$$g_{(4)\mu\nu} = e^{2A} \tilde{g}_{\mu\nu} \quad (7)$$

³For the introduction to anomaly induced effective action in curved space-time (with torsion), see section 5.5 in [12].

and we specify the quantities with $\tilde{g}_{\mu\nu}$ by using $\tilde{\cdot}$. G (\tilde{G}) and F (\tilde{F}) are the Gauss-Bonnet invariant and the square of the Weyl tensor, which are given as ⁴

$$\begin{aligned} G &= R^2 - 4R_{ij}R^{ij} + R_{ijkl}R^{ijkl} \\ F &= \frac{1}{3}R^2 - 2R_{ij}R^{ij} + R_{ijkl}R^{ijkl} , \end{aligned} \quad (8)$$

In the effective action (6), with N scalar, $N_{1/2}$ spinor and N_1 vector fields, b , b' and b'' are

$$b = \frac{(N + 6N_{1/2} + 12N_1)}{120(4\pi)^2} , \quad b' = -\frac{(N + 11N_{1/2} + 62N_1)}{360(4\pi)^2} , \quad b'' = 0 \quad (9)$$

but in principle, b'' may be changed by the finite renormalization of local counterterm in gravitational effective action. As we shall see later, the term proportional to $\left\{b'' + \frac{2}{3}(b + b')\right\}$ in (6), and therefore b'' , does not contribute to the equations of motion (see (29)). For $\mathcal{N} = 4$ $SU(N)$ SYM theory $b = -b' = \frac{N^2-1}{4(4\pi)^2}$. We should also note that W in (6) is defined up to conformally invariant functional, which cannot be determined from only the conformal anomaly. The conformally flat space is a pleasant exclusion where anomaly induced effective action is defined uniquely. However, one can argue that such conformally invariant functional gives next to leading contribution as mass parameter of regularization may be adjusted to be arbitrary small (or large)..

The metric of S_4 with the unit radius is given by

$$d\Omega_4^2 = d\chi^2 + \sin^2 \chi d\Omega_3^2 . \quad (10)$$

⁴We use the following curvature conventions:

$$\begin{aligned} R &= g^{\mu\nu} R_{\mu\nu} \\ R_{\mu\nu} &= R^\lambda_{\mu\lambda\nu} \\ R^\lambda_{\mu\rho\nu} &= -\Gamma^\lambda_{\mu\rho,\nu} + \Gamma^\lambda_{\mu\nu,\rho} - \Gamma^\eta_{\mu\rho}\Gamma^\lambda_{\nu\eta} + \Gamma^\eta_{\mu\nu}\Gamma^\lambda_{\rho\eta} \\ \Gamma^\eta_{\mu\lambda} &= \frac{1}{2}g^{\eta\nu}(g_{\mu\nu,\lambda} + g_{\lambda\nu,\mu} - g_{\mu\lambda,\nu}) . \end{aligned}$$

Here $d\Omega_3^2$ is described by the metric of 3 dimensional unit sphere. If we change the coordinate χ to σ by

$$\sin \chi = \pm \frac{1}{\cosh \sigma} , \quad (11)$$

one obtains

$$d\Omega_4^2 = \frac{1}{\cosh^2 \sigma} (d\sigma^2 + d\Omega_3^2) . \quad (12)$$

On the other hand, the metric of the 4 dimensional flat Euclidean space is given by

$$ds_{4E}^2 = d\rho^2 + \rho^2 d\Omega_3^2 . \quad (13)$$

Then by changing the coordinate as

$$\rho = e^\sigma , \quad (14)$$

one gets

$$ds_{4E}^2 = e^{2\sigma} (d\sigma^2 + d\Omega_3^2) . \quad (15)$$

For the 4 dimensional hyperboloid with the unit radius, the metric is given by

$$ds_{H4}^2 = d\chi^2 + \sinh^2 \chi d\Omega_3^2 . \quad (16)$$

Changing the coordinate χ to σ

$$\sinh \chi = \frac{1}{\sinh \sigma} , \quad (17)$$

one finds

$$ds_{H4}^2 = \frac{1}{\sinh^2 \sigma} (d\sigma^2 + d\Omega_3^2) . \quad (18)$$

We now discuss the 4 dimensional hyperboloid whose boundary is the 3 dimensional sphere S_3 but we can consider the cases that the boundary is a 3 dimensional flat Euclidean space R_3 or a 3 dimensional hyperboloid H_3 . We will, however, only consider the case that the boundary is S_3 since the results for other cases are almost equivalent.

Motivated by (1), (12), (15) and (18), one assumes the metric of 5 dimensional space time as follows:

$$ds^2 = dy^2 + e^{2A(y,\sigma)} \tilde{g}_{\mu\nu} dx^\mu dx^\nu , \quad \tilde{g}_{\mu\nu} dx^\mu dx^\nu \equiv l^2 (d\sigma^2 + d\Omega_3^2) \quad (19)$$

and we identify A and \tilde{g} in (19) with those in (7). Then we find $\tilde{F} = \tilde{G} = 0$, $\tilde{R} = \frac{6}{l^2}$ etc. Due to the assumption (19), the actions in (3), (4), (5), and (6) have the following forms:

$$S_{\text{EH}} = \frac{l^4 V_3}{16\pi G} \int dy d\sigma \left\{ \left(-8\partial_y^2 A - 20(\partial_y A)^2 \right) e^{4A} + \left(-6\partial_\sigma^2 A - 6(\partial_\sigma A)^2 + 6 \right) e^{2A} + \frac{12}{l^2} e^{4A} \right\} \quad (20)$$

$$S_{\text{GH}} = \frac{3l^4 V_3}{8\pi G} \int d\sigma e^{4A} \partial_y A \quad (21)$$

$$S_1 = -\frac{3l^3 V_3}{8\pi G} \int d\sigma e^{4A} \quad (22)$$

$$W = V_3 \int d\sigma \left[b' A \left(2\partial_\sigma^4 A - 8\partial_\sigma^2 A \right) - 2(b + b') \left(1 - \partial_\sigma^2 A - (\partial_\sigma A)^2 \right)^2 \right] . \quad (23)$$

Here V_3 is the volume or area of the unit 3 sphere.

In the bulk, one obtains the following equation of motion from S_{EH} by the variation over A :

$$0 = \left(-24\partial_y^2 A - 48(\partial_y A)^2 + \frac{48}{l^2} \right) e^{4A} + \frac{1}{l^2} \left(-12\partial_\sigma^2 A - 12(\partial_\sigma A)^2 + 12 \right) e^{2A} . \quad (24)$$

Then one finds a solution

$$A = \ln \sinh \frac{y}{l} - \ln \cosh \sigma , \quad (25)$$

which corresponds to the metric of AdS_5 in (1) with (12). There exists also the solution

$$A = \frac{y}{l} + \sigma , \quad (26)$$

which corresponds to (15), and another solution

$$A = \ln \cosh \frac{y}{l} - \ln \sinh \sigma , \quad (27)$$

corresponds to (18). One should note that all the metrics in (25), (26) and (27) locally describe the same spacetime, that is the local region of AdS_5 , in the bulk. As we assume, however, that there is a brane at $y = y_0$, the shapes of the branes are different from each other due to the choice of the metric.

On the brane at the boundary, one gets the following equation:

$$0 = \frac{48l^4}{16\pi G} \left(\partial_y A - \frac{1}{l} \right) e^{4A} + b' (4\partial_\sigma^4 A - 16\partial_\sigma^2 A) - 4(b + b') (\partial_\sigma^4 A + 2\partial_\sigma^2 A - 6(\partial_\sigma A)^2 \partial_\sigma^2 A) . \quad (28)$$

We should note that the contributions from S_{EH} and S_{GH} are twice from the naive values since we have two bulk regions which are connected with each other by the brane. Substituting the bulk solution (25) into (28), one obtains

$$0 = \frac{48l^3}{16\pi G} \left(\coth \frac{y_0}{l} - 1 \right) \sinh^4 \frac{y_0}{l} + 24b' . \quad (29)$$

Note that eq.(29) does not depend on b . The effective tension of the domain wall is given by

$$\sigma_{\text{eff}} = \frac{3}{4\pi G l} \coth \frac{y_0}{l} . \quad (30)$$

As in [9], defining the radius R of the brane in the following way

$$R \equiv l \sinh \frac{y_0}{l} , \quad (31)$$

one can rewrite (29) as

$$0 = \frac{1}{\pi G} \left(\frac{1}{R} \sqrt{1 + \frac{R^2}{l^2}} - \frac{1}{l} \right) R^4 + 8b' . \quad (32)$$

This equation generalizes the corresponding result of ref.[9] for the case when the arbitrary amount of quantum conformal matter sits on de Sitter wall. Adopting AdS/CFT correspondence one can argue that in symmetric phase the quantum brane matter comes due to maximally SUSY Yang-Mills theory.

As we have $b' \rightarrow -\frac{N^2}{4(4\pi)^2}$ in case of the large N limit of $\mathcal{N} = 4$ $SU(N)$ SYM theory, we find

$$\frac{R^3}{l^3} \sqrt{1 + \frac{R^2}{l^2}} = \frac{R^4}{l^4} + \frac{GN^2}{8\pi l^3} , \quad (33)$$

which exactly coincides with the result in [9]. This equation has the unique solution for positive radius which defines brane-world de Sitter Universe (inflation) induced by quantum effects.

On the other hand, if we substitute the solution (26), corresponding to flat Euclidean brane, into (28), we find that (28) is always (independent of y_0) satisfied since $\partial_y A = \frac{1}{l}$ and $\partial_\sigma^2 A = 0$.

If one substitutes (27), which corresponds to the brane with the shape of the hyperboloid, then

$$0 = \frac{48l^3}{16\pi G} \left(\tanh \frac{y_0}{l} - 1 \right) \cosh^4 \frac{y_0}{l} + 24b' . \quad (34)$$

We should note that eq.(34) does not depend on b again. In order that Eq.(34) has a solution, b' must be positive, which conflicts with the case of $\mathcal{N} = 4$ $SU(N)$ SYM theory or usual conformal matter. In general, however, for some exotic theories, like higher derivative conformal scalar⁵, b' can be positive and one can assume for the moment that $b' > 0$ here⁶. Defining the radius R_H of the brane in the following way

$$R_H \equiv l \cosh \frac{y_0}{l} , \quad (35)$$

one can rewrite (29) as

$$0 = \frac{1}{\pi G} \left(\pm \frac{1}{R_H} \sqrt{-1 + \frac{R_H^2}{l^2}} - \frac{1}{l} \right) R_H^4 + 8b' . \quad (36)$$

Hence, we showed that quantum, conformally invariant matter on the wall, leads to the inducing of inflationary 4d de Sitter-brane Universe realized within 5d Anti-de Sitter space (a la Randall-Sundrum[1]). Of course, analytical continuation of our 4d sphere to Lorentzian signature is supposed what leads to ever expanding inflationary brane-world Universe. In 4d QFT (no higher dimensions) such idea of anomaly induced inflation has been suggested long ago in refs.[13]. On the same time the inducing of 4d hyperbolic wall in brane Universe is highly suppressed and may be realized only for exotic conformal matter. The analysis of the role of domain wall CFT to metric fluctuations may be taken from results of ref.[9].

It is interesting to note that our approach is quite general. In particular, it is not difficult to take into account the quantum gravity effects (at least,

⁵Such higher derivative conformal scalar naturally appears in infra-red sector of quantum gravity[14].

⁶For higher derivative conformal scalar $b = -8/120(4\pi)^2$, $b' = 28/360(4\pi)^2$.

on the domain wall). That can be done by using the corresponding analogs of central charge for various QGs which may be taken from beta-functions listed in book [12]. In other words, there will be only QG contributions to coefficients b, b' but no more changes in subsequent equations⁷. The next question which deserves careful investigation is about the (in)stability of such anomaly driven inflation when it evolves to matter dominated Universe. This will be discussed elsewhere.

Acknowledgements. The work by SDO has been supported in part by CONACyT (CP, ref.990356) and in part by RFBR. SDO would like to thank members of Instituto de Fisica de la UG for kind hospitality and Kim Milton for very helpful discussions.

References

- [1] L. Randall and R. Sundrum, *Phys.Rev.Lett.* **83** (1999) 3370, hep-th/9905221; *Phys.Rev.Lett.* **83** (1999) 4690, hep-th/9906064
- [2] H.A. Chamblin and H.S. Reall, hep-th/9903225.
- [3] N. Kaloper, *Phys.Rev.* **D60** (1999) 123506, hep-th/9905210.
- [4] T. Nihei, *Phys.Lett.* **B465** (1999) 81, hep-th/9905487.
- [5] H. Kim and H. Kim, hep-th/9909053.
- [6] J. Garriga and M. Sasaki, hep-th/9912118.
- [7] K. Koyama and J. Soda, hep-th/9912118.
- [8] J. Kim, B. Kyae and H. Min Lee, hep-th/0004005.
- [9] S.W. Hawking, T. Hertog and H.S. Reall, hep-th/0003052.
- [10] S. Nojiri, S.D. Odintsov and S. Zerbini, *Phys.Rev.* **D**, to appear, hep-th/0001192.

⁷For example, for Einstein gravity $b = 611/120(4\pi)^2$ and $b' = -1411/360(4\pi)^2$.

- [11] S. Nojiri, O. Obregon, S.D. Odintsov and S. Ogushi, *Phys.Rev.* **D**, to appear, hep-th/0003148.
- [12] I.L. Buchbinder, S.D. Odintsov and I.L. Shapiro, *Effective Action in Quantum Gravity*, IOP Publishing, Bristol and Philadelphia 1992.
- [13] A. Starobinsky, *Phys.Lett.* **B91** (1980) 99; S.G. Mamaev and V.M. Mostepanenko, *JETP* **51** (1980) 9.
- [14] I. Antoniadis and E. Mottola, *Phys.Rev.* **D45**(1992) 2013; S.D. Odintsov, *Z.Phys.* **C54** (1992) 531.